Vector Differentiation

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Define vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ as

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_n \end{bmatrix}, \tag{1}$$

$$\boldsymbol{y} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ \vdots \\ y_n \end{bmatrix}, \tag{2}$$

then we can have the following relationships

$$\frac{\partial y_i}{\partial \boldsymbol{x}} = \begin{bmatrix} \frac{\partial y_i}{\partial x_1} & \frac{\partial y_i}{\partial x_2} & \frac{\partial y_i}{\partial x_3} & \dots & \frac{\partial y_i}{\partial x_n} \end{bmatrix}, i = 1, 2, 3, \dots, m; \tag{3}$$

$$\frac{\partial \boldsymbol{y}}{\partial x_j} = \begin{bmatrix} \frac{\partial y_1}{\partial x_j} \\ \frac{\partial y_2}{\partial x_j} \\ \frac{\partial y_3}{\partial x_j} \\ \vdots \\ \frac{\partial y_m}{\partial x_j} \end{bmatrix}, j = 1, 2, 3, \dots, n; \tag{4}$$

$$\frac{\partial \boldsymbol{y}}{\partial \boldsymbol{x}} = \begin{bmatrix} \frac{\partial y_1}{\partial x_1} & \frac{\partial y_1}{\partial x_2} & \frac{\partial y_1}{\partial x_3} & \dots & \frac{\partial y_1}{\partial x_n} \\ \frac{\partial y_2}{\partial x_1} & \frac{\partial y_2}{\partial x_2} & \frac{\partial y_2}{\partial x_3} & \dots & \frac{\partial y_2}{\partial x_n} \\ \frac{\partial y_3}{\partial x_1} & \frac{\partial y_3}{\partial x_2} & \frac{\partial y_3}{\partial x_3} & \dots & \frac{\partial y_3}{\partial x_n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \frac{\partial y_m}{\partial x_1} & \frac{\partial y_m}{\partial x_2} & \frac{\partial y_m}{\partial x_3} & \dots & \frac{\partial y_m}{\partial x_n} \end{bmatrix}; \tag{5}$$

$$(\frac{\partial y_i}{\partial \boldsymbol{x}})^T = \frac{\partial y_i}{\partial \boldsymbol{x}^T}, (\frac{\partial \boldsymbol{y}}{\partial x_i})^T = \frac{\partial \boldsymbol{y}^T}{\partial x_i}, (\frac{\partial \boldsymbol{y}}{\partial \boldsymbol{x}})^T = \frac{\partial \boldsymbol{y}^T}{\partial \boldsymbol{x}^T}. \tag{6}$$